00324

HYPERELASTICITY MODELING OF HIGH DAMPING RUBBER AND FINITE ELEMENT SIMULATION

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1. Introduction

The development of high damping rubber (HDR), a new type of elastomer, has opened up a new avenue for application to base isolation bearings as it provides a better energy absorption property than that of other similar materials.

Typically, elastomeric materials exhibit time dependent nonlinear elastic response. However, in case of modeling HDR, plasticity or damage model is additionally needed to simulate permanent set. But even in such situation, a hyperelasticity modeling approach is needed to simulate the rate-independent part of the overall constitutive behavior.

This paper attempts to apply Ogden hyperelastic strain energy function in representing the rate-independent response. Finally, it presents the results obtained from large deformation computation using standard finite element code together with the challenges encountered therein for this new material.

2. Hyperelasticity Modeling

Choice of the approach: In a phenomenological approach, under the assumption of isotropy, hyperelasticity models for elastomers are formulated in terms of strain energy density functions. Numerous strain energy density functions have so far been proposed, which are broadly divided into two groups. Such functions are expressed either in terms of the strain invariants or in terms of the principal stretches. The strain invariant based models are easy to implement in finite element technique, while the stretch based models suggest to be more flexible in fitting experimental data. Again, as far as stress condition in actual isolation bearings under combined action of compression and shear is concerned, stretch based Ogden model has been reported to demonstrate better capability over the other models [1]. Based on this conclusion, the authors have chosen Ogden model for the current application. The following equation presents the strain energy function for Ogden model.

$$\psi(\lambda_i) = \sum_{i=1}^{3}\sum_{j=1}^{3}\frac{c_j}{b_j}(\lambda_i^{b_j} - 1)$$

where, λ_i are the principal stretches and c_j, b_j are the material parameters.

Here, similar to other phenomenological models, the condition $\lambda_1 \lambda_2 \lambda_3 = 1$ incorporates incompressibility into the strain energy function.

Uniaxial Tension Test of HDR and Determination of Material Parameters: The results obtained from the experimental investigation on HDR carried out in the University of Tokyo have been used in this study to verify the applicability of Ogden model. In standard testing method, elastomers in tension are tested on a dumbbell shaped specimen for overcoming the edge effects. The readers are referred to the main publication [2] for details of the test condition.

The six material parameters needed in Ogden model have been determined by a standard method [3]. The parameters have been determined to be C_1 = 0.57 MPa, C_2 = -2.65E-10 MPa, C_3 = 0.06 MPa, b_1 = 2.85, b_2 = 15.00, b_3 = -3.00. Figure 1 presents comparison of the stress-stretch relation with experimental result where a good conformity is observed.

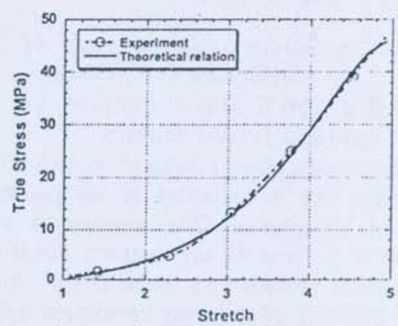

Figure 1. Comparison of stretch-stress relation with experimental data.

3. Finite Element Simulation

The high stiffness of elastomeric materials for volumetric deformations compared to resistance to distortion deformation causes an ill-conditioned system of equations. To overcome the problem, a three field mixed formulation has been used in finite element (FE) simulation of the Ogden model using FEAP, a general - purpose finite element code. The

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incompressibility limit is enforced by augmented Lagrangian method to avoid characteristic ill-conditioning associated with typical penalty methods [4].

Description of Stored Energy Function: For FE computation, the stored energy function (W) decomposed into the volumetric and distortional parts takes the following form

$$W = U(J) + \sum_{i=1}^{3} \psi(\tilde{\lambda}_i, J)$$

where U(J) is the volumetric function, J= det[F], F is the deformation gradient tensor, and $\tilde{\lambda}_i$ is the deviatoric principal stretches defined as $\tilde{\lambda}_i = J^{-\frac{1}{3}}\lambda_i$.

In FE approximation in every element, the hydrostatic pressure field, p_e^h (volumetric part) is described as,

$$p_e^h = \lambda_e^h h'(\Theta_e^h) + \varepsilon\gamma'(\Theta_e^h), \quad \Theta_e^h = \bar{J}_e^h(\varphi^h),$$

$$\gamma(\Theta) = \frac{1}{2}(\Theta^2 - 1) - \log\Theta, \quad h(\Theta) = (\Theta^2 - 1)/\Theta$$

where superscript 'h' and subscript 'e' stand for hydrostatic pressure field description in each element and φ denotes the mapping from the initial to the current particle positions. Θ denotes a kinematic variable representing dilation and prime sign denotes the derivative of a function with respect to Θ. ε is the penalty parameter. λ is the Lagrange multiplier enforcing the condition that Θ=J. Through augmented Lagrangian iteration, as $\varepsilon \rightarrow \infty$; $\Theta \rightarrow 1$ enforces the incompressibility constraint.

Numerical Computation: After a study of mesh sensitivity on deformation prediction, the experimental dumbbell shaped specimen has been discretized into 192 2-D solid elements.

Figure 2 presents the true stress vs. stretch relation obtained from the computation in comparison to experimental observation. The comparison shows marked deviation from the experimental result when the stretch value exceeds 3 i.e. 200% strain. This is a contrast to Figure 1. Here it may be pointed out that, in Ogden strain energy formulation, an absolute incompressibility condition has been assumed but in FE formulation a quasi-incompressible formulation was needed to be used to overcome the ill-conditioning problem associated with incompressibility. This might be a reason behind this deviation. However, further study in this regard is needed to verify this supposition.

Figure 3 presents the stress contours along the loading direction as obtained at 200% strain level. A uniform distribution of stress at the central part of the neck is visible. This agrees with the condition assumed in dumbbell testing.

4. Conclusion

The stretch ratio based Ogden model has been introduced for simulating the rate-independent hyperelastic response of HDR. The constitutive relation adequately represented the experimental result through the determined model parameters. However, in large strain computation a marked deviation has been observed after 200% strain level. Further work is needed to address this issue.

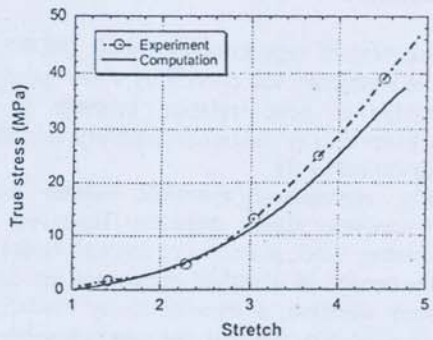

Figure 2. Comparative stretch-stress relation as observed from experiment and FE computation.

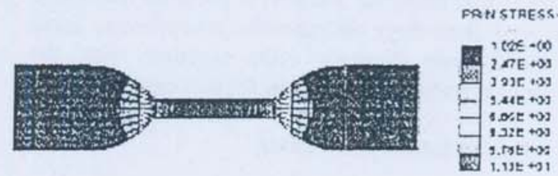

Figure 3. Stress contours along the loading direction.

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