Civil and Environmental Engineering Conference
New Frontiers and Challenges, 8-12 November 1999, Bangkok, Thailand

THICK SHELL FINITE ELEMENTS IN THE ANALYSIS OF HELICOIDAL STAIR SLABS

A. F. M. S. Amin & S. Ahmad
Department of Civil Engineering
Bangladesh University of Engineering and Technology, Dhaka, Bangladesh

ABSTRACT

The complex geometry of helicoidal stair slabs dictates the conventional analytical methods to be based on different idealisations and assumptions. This leads to failure of all the methods in appreciating the beneficial structural behaviour of helicoid. With this background, the paper explores the merits of analysing the structure with curved thick shell finite elements. It reveals that even a coarse mesh of such elements is capable of predicting the deflection behaviour satisfactorily. The load-deflection response obtained from finite element analysis is found to conform with published experimental findings. A case study infers the large over estimation of vertical moment, lateral moment and lateral shear with an under estimation of torsion and thrust in conventional helical girder solution. All these findings evidently indicate the potentials of FE approach over traditional helical girder solution in analysing the structure.

INTRODUCTION

Attractive appearance of the helicoidal stair slab has drawn the attention of the users and architects. For this reason, the structure is now being increasingly used in many buildings. Geometrically, a helicoidal surface is a three dimensional structure in space consisting of a warped surface which is generated by moving a straight line touching a helix so that the moving line is always perpendicular to the axis of the helix. Due to complex geometric configuration of this structure, the conventional methods of analysis are based on different idealisations and assumptions.

In the first approach, Bergman (1956) produced the simplest solution by reducing the helicoid to its horizontal projection and resolving the problem into a fixed ended curved beam. It leads to over estimation of different forces. In the second approach, Holmes (1957), Scordelis (1960) and Morgan (1960) reduced the helicoid to its elastic line having the same stiffness as that of original structure. This simplification neglects the slab action of helicoid and assumes the bending and torsional stiffness of the warped girder as that of a straight beam.

Apart from all these theoretical efforts of finding logical analysis and design procedure for helicoidal stair cases, model tests were also carried out for this structure with a view to substantiating the analytical findings. The first known efforts of testing reinforced concrete helicoidal stairs to failure were made at Asian Institute of Technology (AIT, formerly SEATO Graduate School of Engineering, Bangkok) by Trirojna (1962), Cusens and Trirojna (1964). Two half-scale models having 80-degree central angle were tested under uniform load. The models were scaled down from a prototype, which was designed for the dining hall project of Chulalongkorn University, Thailand. In this study, helical girder approach proposed by Morgan (1960) was used for analysis and design of the model. The first one of these two models was constructed as per the analysis, while in the second model the reinforcement required for resisting computed lateral moment was reduced by 50%. The study estimated that a design based on helical girder solution is sufficient from the safety point of view with a probable load factor of around 5. This load factor, however, seems to be too high from economic design point of view. Apart from these aspects, the experimental work reported that the lateral cross section of the stair slab showed a curved cross section profile under uniformly distributed loading which was described as the evidence of 'slab action' in this structure. All these findings of the model study indicate the necessity of introducing FE approach for studying the behaviour of the structure without any geometric idealisation with a view to achieving some design economy.

With this background, the paper aims to introduce eight node curved thick shell element developed by Ahmad (1968, 1969, 1970) in the analysis of helicoidal stair slabs. In this course, the paper describes the mesh generation technique, effect of the variation of mesh size on deflection computation and also compares the load-deflection response of the structure as obtained by the other researchers through model studies. Finally the paper compares the values of stress resultants as obtained from both helical girder and FE solutions through an illustrative example.

PROTOTYPE STAIR AND STRESS RESULTANTS

The geometry of the prototype stair used in the AIT model study was adopted for the current investigation. Table 1 presents the geometry and loading condition of the half-scale and the full-scale structure.

Table 1 Geometry of the prototype stairs

Parameters	Prototype stairs	
	Full scale	Half scale
Inner radius, m	4.00	2.00
Outer radius, m	6.00	3.00
Height of the stair*, m	3.30	1.65
Waist thickness, m	0.15	0.075
Central angle (on plan), degree	80.00	80.00
Height of the risers, m	0.15	0.075
Uniformly distributed live load, N/m^2	2943.00	2943.00

** Difference of elevation between centre of bottom support and centre of top support*

In general, six stress resultants, i.e. vertical moment, lateral moment, torsion, thrust, lateral shear force and radial horizontal shear force are found on any cross section of a helicoid. The positive directions of these stress resultants have been illustrated in Figure 1.

MESH GENERATION TECHNIQUE

Helicoidal stair slab was divided into a number of elements by equidistant radial lines and helixes. On the horizontal projection they are equidistant radial lines and circular paths respectively. Thus, in this process of division, the eight node parabolic elements require one midside node on each side. Therefore, some radial lines pass through midside nodes and the others through corner nodes. The radial line which passes through the corner nodes as well as midside nodes can be termed as *major radial line*. On the other hand, the radial line which only passes through midside nodes can be termed as *minor radial line*. In a similar concept the *major and minor circular paths* can be defined. The radial line at the bottom can be termed as the *first radial line,* while that at the top can be termed as the *last radial line*. Similarly, the inner most circular path can be defined as the *first circular path* and the outer most one can be defined as the *last circular path*. The first, last radial lines and the circular paths denote the boundary of the structure. Figure 2 shows the scheme of division of the stair into elements.

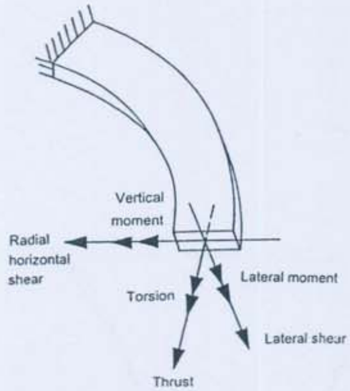

Fig. 1 Stress Resultants

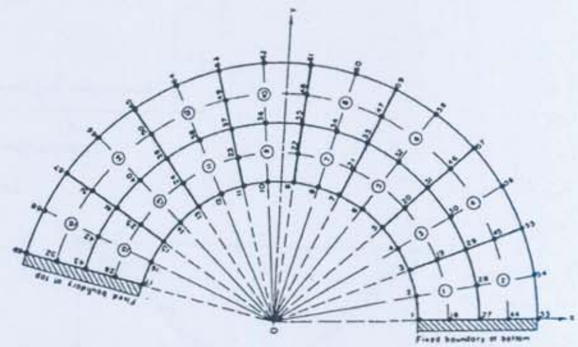

Fig. 2 Mesh Generation Technique

EFFECT OF MESH SIZE ON VERTICAL DISPLACEMENTS

In order to find out an optimum mesh size in using the thick shell element, the full scale stair was analysed with various number of elements along the longitudinal direction and across the transverse direction by varying the number of radial lines and circular paths as well.

At first, the number of elements across the transverse direction was varied from single element to four elements keeping that in the longitudinal direction as constant at eight.

In the second spell, the number of elements along longitudinal direction was varied from two elements to eight elements keeping that in the transverse direction as constant at two. The effect of these variations on the vertical displacements has been presented in Figure 3, which reveals that only two elements in the transverse direction of the stair are sufficient to produce a converging result. In longitudinal direction, 4 elements are found to be sufficient to produce a converging result. The applicability of a coarse mesh (even 2 in transverse direction x 4 in longitudinal direction) in the process of analysis confirms the efficiency of Thick Shell Element that has been used in the present study.

DEFLECTION PROFILE AND LOAD-DEFLECTION RESPONSE

The deflection profile along the inner edge, centre line and outer edge of the prototype stair due to the action of uniformly distributed dead load and live load has been presented in Figure 4 which shows a tendency of tilting in the outward direction with a maximum deflection at the mid span.

In course of application of thick shell elements, the load-deflection response of the stair as obtained in the model studies by Trirojna (1962), Cusens and Trirojna (1964) was compared with that obtained from linear FE analysis. The load-deflection curves obtained from the half scale test of the prototype stair along with the FE findings have been presented in Figure 5.

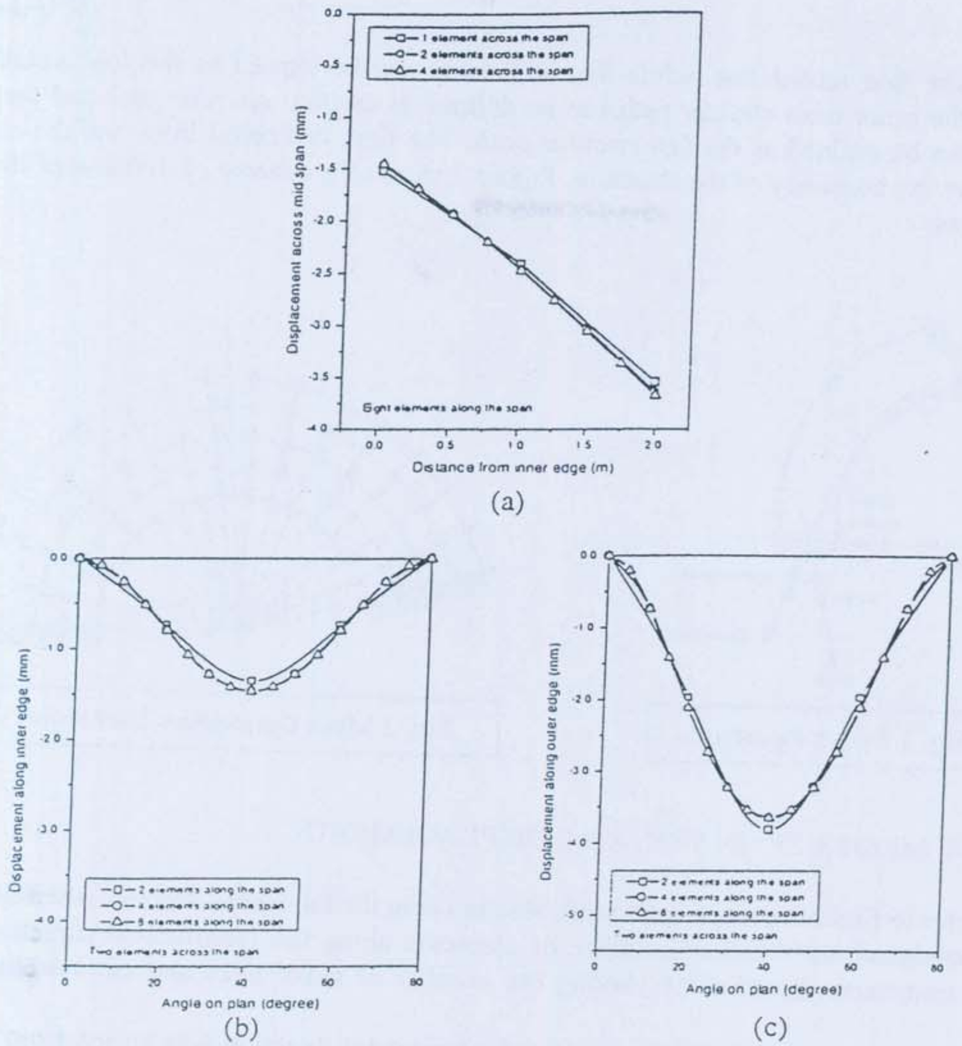

Fig. 3 Effect of mesh size on vertical displacements (a) across the mid span, (b) along inner edge, (c) along outer edge

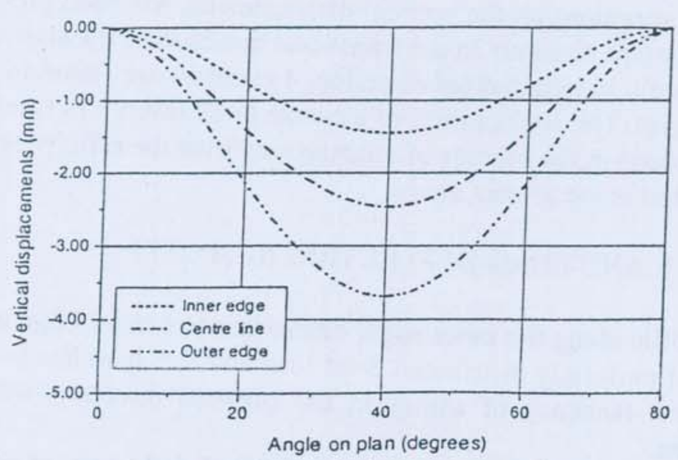

Fig. 4 Vertical displacement profile

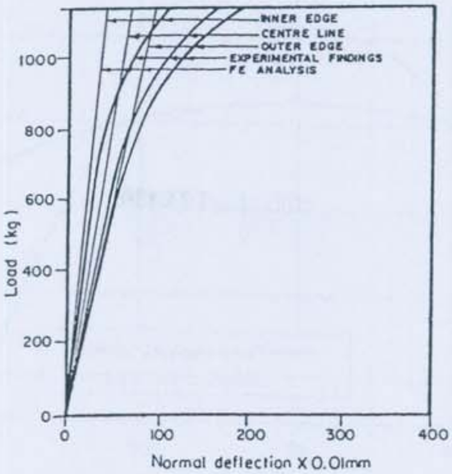

Fig. 5 Comparison of load-deflection response

Due to linear FE analysis, the computed curves are found to be linear in contrast to the non-linear experimental curves which were due to the non-linear concrete properties of the constructed model. However, at low stress level, the FE findings are found to be in good agreement with the experimental findings. In regard to the deflection pattern, it has been noticed that the structure generally tilts outwardly under uniformly distributed loading which is in conformity with the experimental inference.

Apart from these aspects, the experimental work earlier reported that the lateral cross section of the stair slab showed a curved profile under uniformly distributed loading, which was described as the evidence of 'slab action'. However, the lateral displacement profile of the second tested half-scale model of the stair (having 50% of the computed lateral moment resisting reinforcement) showed a milder evidence of this so called 'slab action' (Figure 5). This deviation can now easily be interpreted from the FE findings, where a large over estimation of lateral moment in helical girder approach was noticed (see the next Section). This huge over estimation of lateral moment in helical girder approach (used in the design process of the half scale models) resulted in a massive fabrication of reinforcements along the inner and outer edges forcing the helicoidal slab to behave like an I-section. Evidently, because of the shortcomings of the helical girder solution, the earlier workers failed to give proper interpretation of this deviation.

COMPARATIVE STUDY OF STRESS RESULTANTS

In order to examine the suitability of FE method over helical girder solution in analysing helicoidal stair slabs; the stress resultants obtained from FE solution have been plotted. Figures 6 to 11 present the computed vertical moment, lateral moment, torsion, lateral shear force, radial horizontal shear and thrust force along the span of the prototype stair by the action of dead and live loads. The results obtained from helical girder solutions have simply been superimposed.

The comparison of the curves of this case study clearly reveals that helical girder solution over estimates vertical moment, lateral moment, lateral shear and radial horizontal shear. The extent of over estimation of the support vertical and lateral moments and midspan vertical moment is around five times (Figures 6 and 7), whereas the over estimation of support lateral shear stands over three times (Figure 9). In case of radial horizontal shear, the extent of over estimation is however, found to be quite small (Figure 10). Contrary to all these over estimations, the helical girder approach is found to under estimate torsion and thrust. In these cases the factor of under estimation at the critical locations is nearly two.

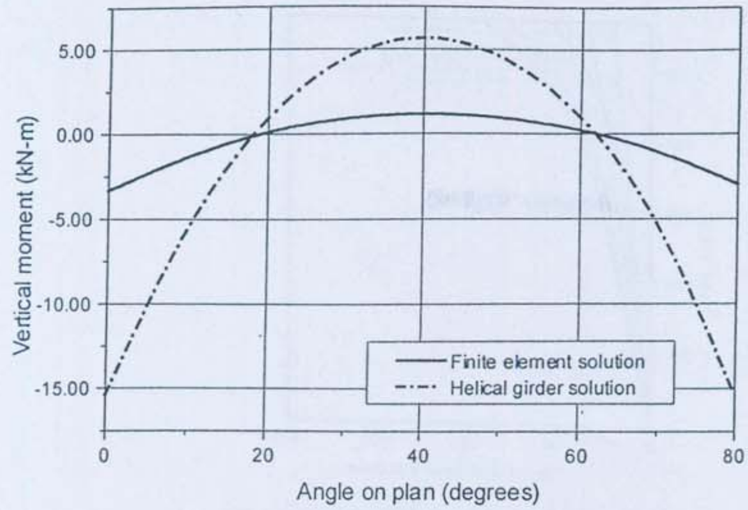

Fig. 6 Vertical moment

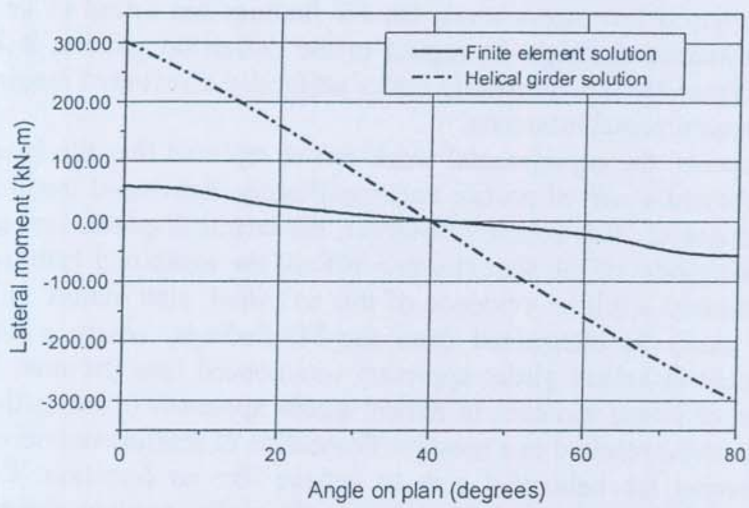

Fig. 7 Lateral moment

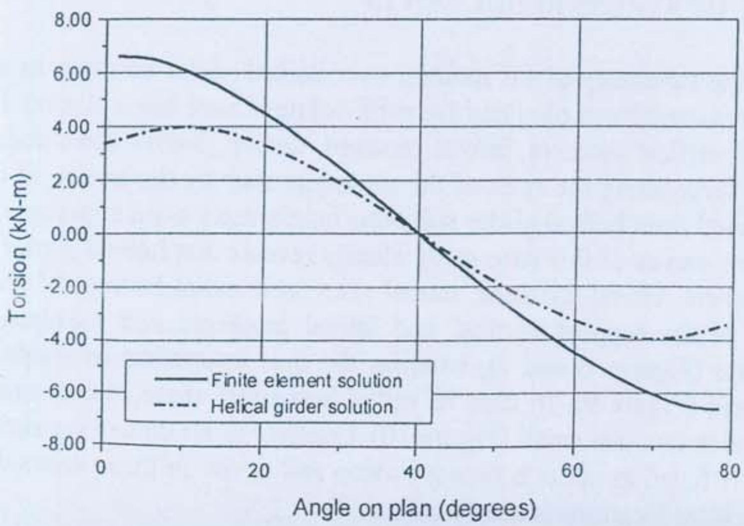

Fig. 8 Torsion

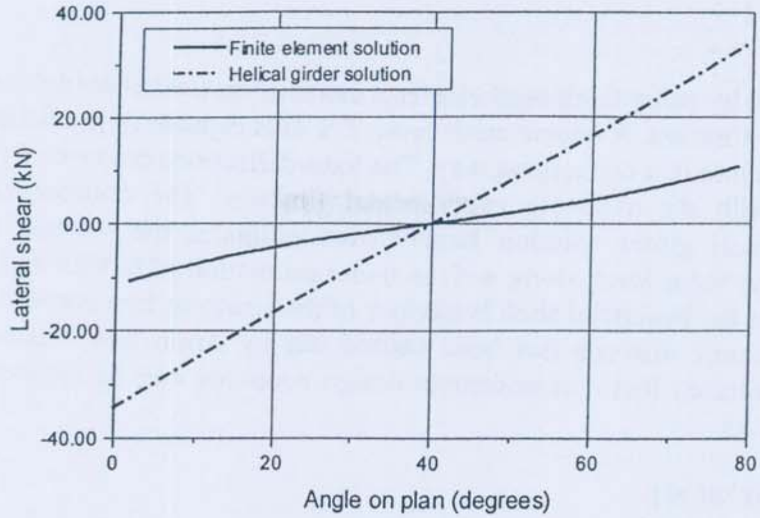

Fig. 9 Lateral shear

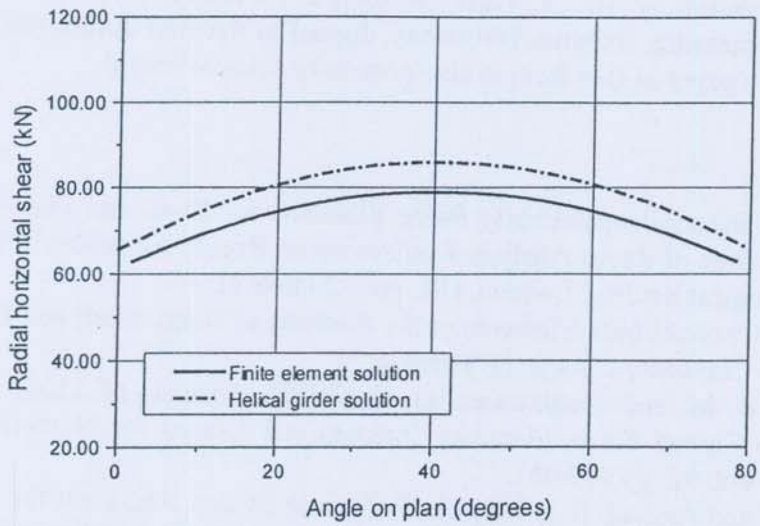

Fig. 10 Radial horizontal shear

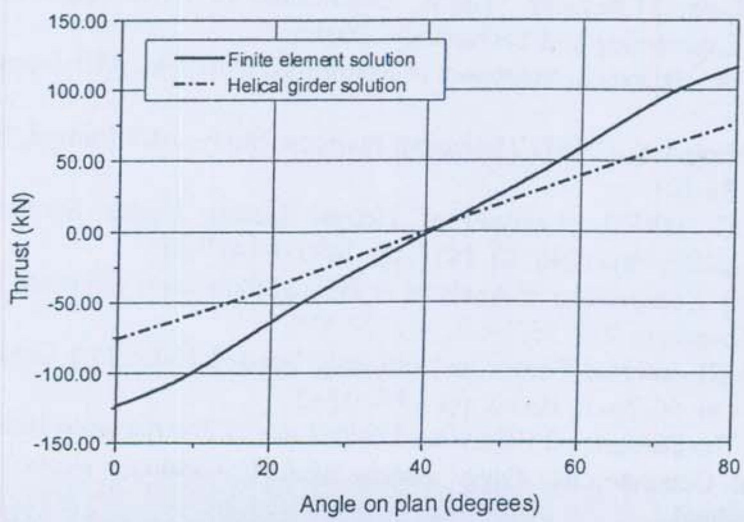

Fig. 11 Thrust

CONCLUSIONS

FE approach by using thick shell elements can analyse the helicoidal stair slabs without any geometric idealisation. A coarse mesh (even 2 x 4) is capable of predicting the deflection profile of the structure in a converging way. The load-deflection response of the structure also compares well with the available experimental findings. The comparative investigation revealed that helical girder solution largely over estimates the vertical moment, lateral moment and lateral shear force along with an under estimation of torsion and thrust. Thus the FE approach takes the beneficial shell behaviour of the structure into account. Based on these findings, an economic analysis has been carried out by Amin and Ahmad (1998), Amin (1998), which indicated that a considerable design economy can be attained with this new approach of analysis.

ACKNOWLEDGEMENT

The authors acknowledge with thanks the Department of Civil Engineering, BUET, Bangladesh for providing facilities to carry out the research described in this paper. The encouragement extended by Dr. Y. Okui (Associate Professor, Department of Civil and Environmental Engineering, Saitama University, Japan) to the first author (on study leave in Japan) to present the paper in this form is also gratefully acknowledged.

REFERENCES

Ahmad, S. (1968). Pseudo-Isoparametric Finite Elements for Shell and Plate Analysis, Joint British Committee of Stress Analysis Conference on Recent Advances in Stress Analysis, Royal Aeronautical Society, London, UK. pp 6-20 to 6-21.

Ahmad, S. (1969). Curved Finite Elements in the Analysis of Solid, Shell and Plate Structures, Ph.D. Thesis, University College of Swansea.

Ahmad, S., Irons, B. M. and Zienkiewicz, O. C. (1970). Analysis of Thick and Thin Shell Structures by Curved Finite Elements, International Journal for Numerical Methods in Engineering, Vol. 02, pp 419-451.

Amin, A. F. M. S., and Ahmad, S. (1998). An Economic Design Approach for Helicoidal Stair Slabs Based on Finite Element Analysis, Advances in Civil and Structural Engineering Computing for Practice, Civil-Comp Ltd., Edinburgh, UK. pp. 241-246.

Amin, A. F. M. S. (1998). Improved Design Rationale for Helicoidal Stair Slabs Based on Finite Element Analysis, M.Sc.Engg. Thesis, Department of Civil Engineering, Bangladesh University of Engineering and Technology, Dhaka.

Bergman, V.R. (1956). Helicoidal Staircases of Reinforced Concrete, ACI Journal, Vol. 28, 403-412.

Cusens, A.R. and Trirojna, S. (1964). Helicoidal Staircase Study, ACI Journal, Proceedings, Vol. 61, No. 1, pp. 85-101.

Holmes, Alan, M.C. (1957). Analysis of Helical Beams Under Symmetrical Loading, Proceedings, ASCE, Paper No. ST 1437, pp. 1437-1-1437-37.

Morgan, V.A. (1960). Comparison of Analysis of Helical Staircases, Concrete and Construction Engineering (London), Vol. 55, No.3, pp. 127-132.

Scordelis, A.C. (1960). Internal Forces in Uniformly Loaded Helicoidal Girder, ACI Journal, Proceedings, Vol. 56, No.6, Part-2, pp. 1491-1502.

Trirojna, S. (1962). The Design and Behaviour Under Load of Fixed-Ended Helicoidal Staircases of Reinforced Concrete, M. Engg. Thesis, SEATO Graduate School of Engineering, Bangkok, Thailand.